

Holographic bounds and Higgs inflation

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Abstract

In a recently proposed scenario for primordial inflation, where the Standard Model (SM) Higgs boson plays a role of the inflation field, an effective field theory (EFT) approach is the most convenient for working out the consequences of breaking of perturbative unitarity, caused by the strong coupling of the Higgs field to the Ricci scalar. The domain of validity of the EFT approach is given by the ultraviolet (UV) cutoff, which, roughly speaking, should always exceed the Hubble parameter in the course of inflation. On the other hand, applying the trusted principles of quantum gravity to a local EFT demands that it should only be used to describe states in a region larger than their corresponding Schwarzschild radius, manifesting thus a sort of UV/IR correspondence. We consider both constraints on EFT, to ascertain which models of the SM Higgs inflation are able to simultaneously comply with them. We also show that if the gravitational coupling evolves with the scale factor, the holographic constraint can be alleviated significantly with minimal set of canonical assumptions, by forcing the said coupling to be asymptotically free.

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In search for the most elegant model for primordial inflation, keeping at the same time a minimal particle content, a recently proposed model in which the role of the inflaton field is played by the Higgs boson of the SM, certainly leaps out [1]. The original model, with no further degrees of freedom beyond the SM ones, requires an unnaturally large coupling between the Higgs field and the scalar curvature, typically $\xi \sim 10^4$, to produce successful inflation. However, the same coupling becomes responsible for the presence of higher dimensional operators (inducing in so doing also some scale Λ), whose influence could no longer be ignored at energies near Λ . Thus, at energies $E \simeq \Lambda$ a perturbative treatment of the inflatory dynamics ceases to be valid because of violation of the tree-level unitarity (or, alternatively, because of entering the strong coupling regime). The cutoff Λ turns out to be dependent of the background value of the Higgs field [2], and since it could be quite large during inflation, there is a hope that the intrinsic energy scale involved in the physical processes during inflation be less than Λ , thereby necessitating no understanding on the UV completion of the theory. Heuristically, the EFT approach, being thus insensitive to new physics above Λ at tree level [2], suffices if $\Lambda \gg H_{inf}$, where H_{inf} is the Hubble parameter during inflation. The question of whether the above scenario is free from unitarity problems has been discussed extensively recently [3–11]. For earlier attempts, see [12].

Especially promising in this respect has appeared the new model of the SM Higgs inflation, featuring the unique nonminimally derivative coupling of the Higgs boson to gravity [13]. However, together with the original model [1], this one too was shown to suffer from unitarity problems, meaning that some new physics is likely to show up in the inflatory dynamics [14, 15]. Finally, using the Palatini formulation of gravity instead of the metric one, the original model of Higgs inflation can be made a UV-safe, as the UV cutoff is always larger than the Hubble rate [16].

In the present paper, we would like to draw attention to another constraint which could possibly endanger the EFT approach of the SM Higgs inflation - the holographic bound. Our main observation here is that an EFT underlying inflation should be valid in the whole Hubble volume during inflation. However, whenever EFT is to be valid in volumes $\gg \Lambda^{-3}$, this might jeopardize the absolute bound on the entropy of a black hole. Indeed, for an EFT in a box of size L (providing an IR cutoff) the entropy scales extensively, $S_{EFT} \sim L^3 \Lambda^3$, and therefore there is always a sufficiently large volume for which S_{EFT} would exceed the absolute Bekenstein-Hawking bound $S_{BH} \sim L^2 M_{Pl}^2$, where M_{Pl} is the Planck mass. To

prevent such an undesirable scenario, a kind of UV/IR correspondence, $L\Lambda^3 \lesssim M_{Pl}^2$ [17], is to be imposed on a theory to assure its validity in arbitrarily large volumes. However, at saturation, this bound means that an EFT should also be capable to describe systems containing black holes, since it necessarily includes many states with Schwarzschild radius much larger than the box size. There are however arguments for why an ordinary local EFT appears unlikely to provide an adequate description of any systems containing black holes, which we list below.

Having become a trusted principle of quantum gravity and even part of the mainstream after the Maldacena's discovery of AdS/CFT duality [18], the holographic principle [19, 20] virtually deprives any local EFT from being capable to describe states in a region smaller than their corresponding Schwarzschild radius [17]. For instance, as a closely related concept emerging from the holographic principle, the black hole complementarity [21] relativizes the very notion of location a certain phenomenon takes place at, something that an ordinary local EFT certainly is not capable to describe. On relying on locality, as an important property of quantum field theory, we get that quantum evolution seemingly ceases to be unitary in black hole backgrounds (operators at space-like points no longer commute) [21]. Also, near the horizon of a black hole, the momenta of infalling particles as seen by an outside observer grow exponentially with time [22], a phenomenon certainly at odds with effective, low-energy description. Thus, summing up, ordinary EFTs may not be valid to describe those states already collapsed to a black hole, implying a constraint much more stringent than that obtained from the entropy of a black hole, $\Lambda^4 \lesssim L^{-2}M_{Pl}^2$ [17].¹ Note, though, that this latter constraint is not absolute in character as the former one, obtained from the requirement that for an EFT in an arbitrarily-sized box the entropy should not exceed the entropy of the same-sized black hole. Still, it is based on the generally accepted view (supported, among others, with the bunch of strong arguments as given above) that black holes and their interaction with quantum fields are not likely to be described by any EFT. So, we deem this prevailing wisdom as a solid foundation for the latter, more stringent constraint, which we are going to employ in this paper.

The EFT approach of inflatory dynamics thus requires that the Hubble length always

¹ This relates to a huge gap in entropy between a system on the brink of experiencing a sudden collapse to a black hole ($S \sim S_{BH}^{3/4}$) and the black hole itself, with both systems having the same size and energy [17, 23, 24].

exceeds the Schwarzschild radius of states encompassed by the Hubble volume, $H_{inf}^{-1} \gtrsim R_S$, or equivalently ²

$$\Lambda^4 \lesssim H_{inf}^2 \bar{M}_{Pl}^2, \quad (1)$$

where \bar{M}_{Pl} is the reduced Planck mass. Note that (1) bounds Λ from above, whereas the (heuristic) constraint coming from violation of unitarity gives the lower bound for Λ . In combination, both constraints require $\Lambda \ll \bar{M}_{Pl}$. However, we study below more precisely some Higgs inflation models to see if they comply to (1). ³

The original (toy) model of Higgs inflation [1] is given by the action

$$S = \int d^4x \sqrt{-g} \left[\frac{\bar{M}_{Pl}^2}{2} R + \xi \mathcal{H}^\dagger \mathcal{H} \mathcal{R} + \mathcal{L}_{\mathcal{SM}}^s \right], \quad (2)$$

where R is the Ricci scalar, \mathcal{H} is the SM Higgs doublet and $\mathcal{L}_{\mathcal{SM}}^s$ is the scalar part of the SM Lagrangian. As already stated, the Higgs-Ricci coupling of order of $\xi \sim 10^4$ is needed to obtain successful inflation. The background Higgs field during the inflationary period, $\bar{\phi} \gg \bar{M}_{Pl} \xi^{-1/2}$, gives rise to an UV cutoff in the Jordan frame $\Lambda^J \simeq \xi^{1/2} \bar{\phi}$ [2]. On the other hand, the Hubble parameter in the Jordan frame reads $H_{inf}^J \simeq \lambda^{1/2} \xi^{-1/2} \bar{\phi}$, where λ is the Higgs quartic coupling entering $\mathcal{L}_{\mathcal{SM}}^s$. Plugging all this back into (1), one arrives at $\xi \ll \lambda^{1/2}$. Considering the current experimental bounds on λ of order of $O(10^{-1})$ [25], one plainly sees that the original (toy) model of the SM Higgs inflation does not respect the holographic bound.

Next, we examine within the same model of the Higgs-driven inflation whether the reheating phase immediately after inflation does respect the holographic bound (1). When the inflatory epoch relinquishes its dominance to a matter dominated stage featuring scalar field oscillations, we have $\xi^{-1} \bar{M}_{Pl} \ll \bar{\chi} < \bar{M}_{Pl}$ [2, 26], where $\bar{\chi}$ is the canonically normalized scalar field in the Einstein frame. During this stage the scalar potential is essentially quadratic, and the Hubble parameter is approximately given as $H_{reh}^E \simeq \lambda^{1/2} \xi^{-1} \bar{\chi}$. With $\Lambda^E \simeq \bar{\chi}$ [2, 26], one finds by using (1) that $\bar{\chi} \lesssim \lambda^{1/2} \xi^{-1} \bar{M}_{Pl}$. Again, the holographic bound is not respected.

The addition of the gauge bosons in the original (toy) model of Higgs inflation results in lower cutoffs both in the inflatory and reheating epoch [2]. In the Einstein frame, the

² Throughout this paper we aim not to keep records of numerical factors (of order $O(1)$).

³ The Hubble parameter in (1) gives rise to a cosmological event horizon and therefore to a cosmological black hole, which, from our perspective, lies beyond the horizon, leaving thus our considerations unaffected.

above cutoffs are replaced with $\Lambda^E \simeq \xi^{-1/2} \bar{M}_{Pl}$ for the inflatory epoch, and with $\Lambda^E \simeq \xi^{-1/2} \bar{\chi}^{1/2} \bar{M}_{Pl}^{1/2}$ for the reheating phase. For the inflatory phase, Eq. (1) gives $\lambda \gtrsim 1$, which means that even upon inclusion of the SM fields, the holographic bound is not respected. In the reheating phase $\xi^{-1} \bar{M}_{Pl} \ll \bar{\chi} < \bar{M}_{Pl}$, the requirement (1) gives again $\lambda \gtrsim 1$. This shows that the inclusion of the gauge bosons does not reduce the cutoffs sufficiently, so that (1) is not obeyed by the original model.

The unitarity violation scale is defined by the minimal cutoff read out of all higher dimensional operators. In the original model of Higgs inflation, the unitarity violation scale in the Jordan frame is given by the cutoff read out of the lowest (5-dimensional) operator, once one expands about the background values. It is just the opposite for the holographic bound, where the cutoff appears as a lower bound in Eq. (1). This means that inclusion of higher dimensional operators could aggravate the holographic bound. Indeed, it can be shown that the cutoffs associated to higher dimensional operators ($n > 5$) get further amplified by the background value of the scalar field, the highest energy scale during inflation. The original model of Higgs inflation is undoubtedly at odds with the holographic bound.

The new model of Higgs inflation, promoting the coupling of the Higgs boson to the Einstein tensor instead to the Ricci scalar (at the same time keeping the minimal SM content), is given by the action [13]

$$S = \int d^4x \sqrt{-g} \left[\frac{\bar{M}_{Pl}^2}{2} R + \frac{1}{2} (g^{\mu\nu} - \omega^2 G^{\mu\nu}) \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \phi^4 \right], \quad (3)$$

where $G^{\mu\nu} = R^{\mu\nu} - (R/2)g^{\mu\nu}$ is the Einstein tensor, ϕ is the real Higgs field and ω^{-1} is some mass scale. The UV cutoff of the EFT approach, emerged from violation of the tree-level unitarity in the course of inflation, is given in the Jordan frame as $\Lambda^J \simeq (\bar{H}_{inf}^{J2} \bar{M}_{Pl})^{1/3}$ [13, 14]. With $\bar{H} \simeq \lambda^{1/2} \bar{M}_{Pl}^{-1} \bar{\phi}^2$ [13], the holographic bound (1) gives $\bar{\phi} \lesssim \lambda^{-4} \bar{M}_{Pl}$. The model has been tested against the most recent WMAP data, to give the allowed size for the background field in a small range near $\bar{\phi} \sim 10^{-2} \bar{M}_{Pl}$ [27]. Although it safely passes the holographic test with the lowest cutoff coming from the 7-dimensional operator, the inclusion of higher dimensional operators could again vitiate this agreement. Indeed, although with the cutoff read out of the 8-dimensional operator, $\Lambda^J \simeq (\bar{H}_{inf}^{J2} \bar{M}_{Pl}^2)^{1/4}$, the holographic bound stays at least marginally respected, it becomes violated by the posterior higher dimensional operator. The cutoff read out of 9-dimensional operator, $\Lambda^J \simeq (\bar{H}_{inf}^{J2} \bar{M}_{Pl}^3)^{1/5}$, implies, when

plugged in (1), $\bar{\phi} \gtrsim \lambda^{-1/4} \bar{M}_{Pl}$. Clearly, this is at odds with [27]. On top of that, it was argued recently [14] that the model is still plagued with unitarity problems.

Recently, it was shown[16] by using the Palatini instead of metric formulation of gravity, that unitarity issues can be alleviated in the Higgs inflation scenario [1], retaining at the same time the minimal particle content. In particular, it was found that the ratio Λ/H can be made greater in the Palatini formulation of gravity. However, this would obviously exacerbate compliance with the holographic bound.

Another interesting conjecture [14], serving to increase the ratio Λ/H , is to promote the gravitational coupling to a running quantity, with the property of being asymptotically safe. Heuristically, while the original model [1] is not affected to a great extent since the ratio Λ/H does not contain the \bar{M}_{Pl} factor in the inflatory period, the same ratio in the new model [13] grows as $\bar{M}_{Pl}^{2/3}$.⁴ At the same time the holographic bound is alleviated in both models.

Note that the amount of running in the UV one can expect depends on the transfer of energy between the various components at a particular cosmological epoch. In particular, when G is not static and owing to the Bianchi identity satisfied by the Einstein tensor on the *lhs* of the Einstein equation, it is a quantity $GT_{total}^{\mu\nu}$, where $T_{total}^{\mu\nu}$ is the total energy-momentum tensor, which is actually conserved. Here $T_{total}^{\mu\nu}$ counts all forms of energy densities (radiation, matter, vacuum, ...). Owing to the fact that the total energy-momentum tensor is no longer conserved when G is running, the energy transfer between various forms of energy densities is now also controlled by the scaling of the gravitational coupling. This leads to the generalized equation of continuity, which in the FLRW metric takes the form

$$\dot{G}(\rho_\Lambda + \rho_m) + G\dot{\rho}_\Lambda + G(\dot{\rho}_m + H\alpha_m\rho_m) = 0 . \quad (4)$$

Here overdots denote time derivative, $\alpha_m \equiv 3(1 + \omega_m)$ introduces the EOS for matter $p_m = \omega_m\rho_m$, and $p_\Lambda \simeq -\rho_\Lambda$.

At the stage of inflation it is naturally to set $\rho_m \simeq 0$, so G varies only at expense of the inflaton potential energy. However, the slow-roll approximation precludes any discernible running at this stage. Also, from this model-independent point of view, it is impossible to fix the gradient of G .

⁴ Of course, the parameter $\bar{M}_{Pl} \sim G^{-1}$ is infrared-free.

This situation is however changed upon considering the subsequent cosmological epoch, where matter is being created. With canonical assumption that matter is conserved, assuming also some leftover vacuum energy in the spirit of the holographic bound (1), $\rho_\Lambda(\mu) \simeq c\mu^2 G^{-1}(\mu)$,⁵ where μ is the energy scale and $c \lesssim 1$, we obtain from (4) that

$$\mu = -\frac{G'(\mu)\rho_m}{2c}. \quad (5)$$

Some scaling properties of G as implied by holography can be easily inferred from (5). Namely, from the requirement of the positivity of the scale μ , $\mu > 0$, it is seen that $G'(\mu) < 0$, which consequently means that $\dot{G}(t) > 0$, *i. e.*, $G(t)$ increases as a function of cosmic time. Such a scale dependence implies that the coupling G is asymptotically free; a feature exhibited, for instance, by higher-derivative quantum gravity models at the 1-loop level [29–31] or in conformal field theory in curved spacetime [32]. More generally, this fits with the approach to quantum gravity, based on the existence of a nontrivial fixed point for the dimensionless gravitational coupling in the UV (for a review see [33]). We find it very appealing to point to the asymptotically free regime for G in a quite model independent way, relying solely on the holographic principle. Consequently, the holographic bound (1) is alleviated in this regime.

In conclusion, we have noted that besides the restriction on the UV cutoff coming from the scale of violation of the tree-level unitarity, the effective field theory approach to the Higgs inflation should not also be used to describe states inside a black hole. Since the latter constraint gives the upper limit on the UV cutoff, we have shown that both restrictions are, in general, very hard to reconcile. We have noted that both restrictions can be alleviated in the scenario of asymptotically safe gravity. We have demonstrated, by relying on the holographic principle only and using a set of reasonable assumptions, how this goal can be achieved in a model independent way.

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⁵ This strongly resembles the model for holographic dark energy [28], a stuff prevailing at present but suppressed at earlier cosmological epochs.

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